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FUZZY SUPRA β-OPEN SETS

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ABSTRACT

In this paper fuzzy supra β-open sets and fuzzy supra β-closed sets are introduced and certain properties and relations of fuzzy supra β-open and fuzzy supra β-closed sets are investigated.

KEYWORDS: Fuzzy supra topological space, fuzzy supra β-open set, fuzzy supra β-closed set.

INTRODUCTION

In 1983, Abd El. Monsef M.E et al., [1] introduced the notion of β-open sets. In 1983, Mashhour, A.S et al., [11] introduced the concept of supra topological spaces and in 1987, Abd El-Monsef M.E et al., [2] introduced fuzzy supra topological spaces. In 1999, Balasubramanian, G [5] introduced the concept of fuzzy β-open sets. In 2013, Mahmood, B.K [10] introduced supra β-open sets.

In this paper fuzzy supra β-open sets and fuzzy supra β-closed sets are introduced. In section 2 of this paper preliminary definitions and properties regarding fuzzy sets and fuzzy supra sets are given. In section 3 of this paper the concept of fuzzy supra β-open sets and fuzzy supra β-closed sets are introduced and certain properties and relations of fuzzy supra β-open and fuzzy supra β-closed sets are investigated.

Preliminary Definitions

Throughout the paper X denotes a non empty set.

Definition: 2.1 [7]

A fuzzy set in X is a map f: X → [0, 1] = I. The family of fuzzy sets of X is denoted by I^X.

Following are some basic operations in fuzzy sets in X. For the fuzzy sets f and g in X,

1) f = g if and only if f(x) = g(x) for all x ϵ X
2) f ≤ g if and only if f(x) ≤ g(x) for all x ϵ X
3) (f ∨ g)(x) = max { f(x),g(x) } for all x ϵ X
4) (f ∧ g)(x) = min { f(x),g(x) } for all x ϵ X
5) $f^c(x)$ = 1- f(x) for all x ϵ X here f^c denotes the complement of f.
6) For a family { f_λ/ λϵΛ } of fuzzy sets defined on a set X

   $(\bigvee_{\lambda\in\Lambda} f_\lambda)(x) = \bigvee_{\lambda\in\Lambda} (f_\lambda(x))$

   $(\bigwedge_{\lambda\in\Lambda} f_\lambda)(x) = \bigwedge_{\lambda\in\Lambda} (f_\lambda(x))$
7) For any α ϵ I, the constant fuzzy set **α** in X is a fuzzy set in X defined by **α**(x) = α for all x ϵ X.

0 denotes null fuzzy set in X and **1** denotes universal fuzzy set in X.

Definition: 2.2 [7]

A fuzzy topological space is a pair (X, δ) where X is a nonempty set and δ is a family of fuzzy set on X satisfying the following properties:

1) The constant fuzzy sets **0** and **1** belong to δ.
2) f, g ϵ δ implies f ∧ g ϵ δ.
3) f_λ ϵ δ for each λ ϵ Λ implies $(\vee_{\lambda\epsilon\Lambda} f_\lambda)$ϵδ.

Then δ is called a fuzzy topology on X. Every member of δ is called fuzzy open set. The complement of a fuzzy open set is called fuzzy closed set.

Definition: 2.3 [7]

The closure and interior of a fuzzy set f ϵ I^X are defined respectively as

cl (f) = ∧ {g / g is a fuzzy closed set in X and f ≤ g}

int (f)= v {g / g is a fuzzy open set in X and g ≤ f }

Clearly cl(f) is the smallest fuzzy closed set containing f and int(f) is the largest fuzzy open set contained in f.

Definition: 2.4 [15]

A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied:

1) The constant fuzzy sets **0** and **1** belong to δ^*.
2) $f_\lambda \epsilon\, \delta^*$ for each λ ϵ Λ implies $(\vee_{\lambda\epsilon\Lambda} f_\lambda)\epsilon\, \delta^*$.

The pair (X, δ^*) is called a fuzzy supra topological space. The elements of δ^* are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition: 2.5 [15]

Let (X, δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior of f defined respectively as

$cl^*(f)$ = ∧ { g / g is a fuzzy supra closed set in X and f ≤ g}

$int^*(f)$ = v { g / g is a fuzzy supra open set in X and g ≤ f}

Definition: 2.6 [15]

Let (X, δ) be a fuzzy topological space and δ^* be a fuzzy supra topology on X .
We call δ^* a fuzzy supra topology associated with δ if δ ≤ δ^*

Remark: 2.7 [15]

1) The fuzzy supra closure of a fuzzy set f in a fuzzy supra topological space is the smallest fuzzy supra closed set containing f.
2) The fuzzy supra interior of a fuzzy set f in a fuzzy supra topological space is the largest fuzzy supra open set contained in f.
3) If (X, δ^*)is an associated fuzzy supra topological space with the fuzzy topological space (X, δ) and f is any fuzzy set in X, then

   $int (f) \leq int^*(f) \leq f \leq cl^*(f) \leq cl (f)$

Theorem: 2.8 [15]

For any two fuzzy sets f and g in a fuzzy supra topological space(X, δ^*),

1) f is a fuzzy supra closed set if and only if $cl^*(f) = f$.
2) f is a fuzzy supra open set if and only if $int^*(f) = (f)$.
3) f ≤ g implies $int^*(f) \leq int^*(g)$ and
   $cl^*(f) \leq cl^*(g)$
4) $cl^*(cl^*(f)) = cl^*(f)$ and $int^*(int^*(f)) = int^*(f)$.
5) $cl^*(f \vee g) \geq cl^*(f) \vee cl^*(g)$
6) $cl^*(f \wedge g) \leq cl^*(f) \wedge cl^*(g)$
7) $int^*(f \vee g) \geq int^*(f) \vee int^*(g)$

8) $int^*(f \wedge g) \leq int^*(f) \wedge int^*(g)$
9) $cl^*(f^c) = [int^*(f)]^c$
10) $int^*(f^c) = [cl^*(f)]^c$

Definition: 2.9 [15]
Let (X, δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra semi open set if

$$f \leq cl^*(int^*(f))$$

The complement of a fuzzy supra semi open set is called a fuzzy supra semi closed set.

Definition: 2.10 [9]
Let (X, δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra preopen set if

$$f \leq int^*(cl^*(f))$$

The complement of a fuzzy supra preopen set is called a fuzzy supra preclosed set.

FUZZY SUPRA β-OPEN SETS

Definition: 3.1
Let (X,δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra β-open set if

$$f \leq cl^*[int^*(cl^*(f))]$$

The complement of a fuzzy supra β-open set is called a fuzzy supra β-closed set.

Theorem: 3.2
Every fuzzy supra open set is fuzzy supra β-open set.

Proof:
Let f be a fuzzy supra open set in (X, δ^*)

$\therefore int^*(f) = f$ ---------------------------------(1)

By the property of fuzzy supra interior, "$f \leq g$ implies that $int^*(f) \leq int^*(g)$"

Since $f \leq cl^*(f)$,

$int^*(f) \leq int^*[cl^*(f)]$

$\therefore f \leq int^*[cl^*(f)]$ (from (1)) -----------(2)

By the property of fuzzy supra closure, "$f \leq g$ implies that $cl^*(f) \leq cl^*(g)$"

From (2)

$cl^*(f) \leq cl^*(int^*(cl^*(f)))$

$\therefore f \leq cl^*(int^*(cl^*(f)))$ ($\because f \leq cl^*(f)$).

Theorem: 3.3
Every fuzzy supra closed set is fuzzy supra β-closed set.

Proof:
Let f be a fuzzy supra closed set in (X, δ^*)

$\therefore cl^*(f) = f$--------------------------------(3)

By the property of fuzzy supra closure, "$f \leq g$ implies that $cl^*(f) \leq cl^*(g)$"

Since $int^*(f) \leq f$

$\therefore cl^*[int^*(f)] \leq cl^*(f)$

$\therefore cl^*[int^*(f)] \leq f$ (from (3))--------------(4)

By the property of fuzzy supra interior, "$f \leq g$ implies that $int^*(f) \leq int^*(g)$"

$\therefore int^*[cl^*(int^*(f))] \leq int^*(f)$

$\therefore int^*[cl^*(int^*(f))] \leq f$ ($\because int^*(f) \leq f$)

Theorem: 3.4
Every fuzzy supra preopen set is fuzzy supra β-open set.

Proof:

Let f be a fuzzy supra preopen set in (X,δ^*), then

$f \leq int^*(cl^*(f))$--------------------(1)

By the property of fuzzy supra closure,

"$f \leq g$ implies $cl^*(f) \leq cl^*(g)$"

∴ From (1)

$cl^*(f) \leq cl^*(int^*(cl^*(f)))$

∴ $f \leq cl^*(int^*(cl^*(f)))$ $(\because f \leq cl^*(f))$.

Theorem: 3.5

1) Finite union of fuzzy supra β-open sets is a fuzzy supra β-open set.
2) Finite intersection of fuzzy supra β-closed sets is a fuzzy supra β-closed set.

Proof:

1) Let f and g be two fuzzy supra β-open sets. Then

$f \leq cl^*(int^*(cl^*(f)))$

and $g \leq cl^*(int^*(cl^*(g)))$

implies $f \vee g \leq cl^*(int^*(cl^*(f \vee g)))$---------------(1)

∴ Finite union of fuzzy supra β-open sets is a fuzzy supra β-open set

2) From (1)

$f \vee g \leq cl^*(int^*(cl^*(f \vee g)))$

By taking complement

$(f \vee g)^c \geq [cl^*(int^*(cl^*(f \vee g)))]^c$

$f^c \wedge g^c \geq int^*[(int^*(cl^*(f \vee g)))^c]$

$\geq int^*(cl^*[(cl^*(f \vee g))^c])$

$\geq int^*(cl^*(int^*[(f \vee g)^c]))$

$\geq int^*(cl^*(int^*(f^c \wedge g^c)))$

∴ Finite intersection of fuzzy supra β-closed sets is always a fuzzy supra β- closed set.

Definition: 3.6

Let (X, δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra β-closure and fuzzy supra β-interior of f defined respectively as

$\beta cl^*(f) = \wedge \{ g / g$ is a fuzzy supra β-closed set in X and $f \leq g\}$

$\beta int^*(f) = \vee \{ g / g$ is a fuzzy supra β-open set in X and $g \leq f\}$

Remark: 3.7

It is obvious that $\beta int^*(f)$ is a fuzzy supra β-open set and $\beta cl^*(f)$ is a fuzzy supra β-closed set.

Theorem: 3.8

For any fuzzy set f in a fuzzy supra topological space(X, δ^*),

1) $[\beta int^*(f)]^c = \beta cl^*(f^c)$
2) $[\beta cl^*(f)]^c = \beta int^*(f^c)$

Proof:

1) Consider

$\beta int^*(f) = \vee \{ g / g$ is a fuzzy supra β-open set in X and $g \leq f\}$

$[\beta int^*(f)]^c = 1 - \vee \{ g / g$ is a fuzzy supra β-open set in X and $g \leq f\}$

$= \wedge \{ g^c / g^c$ is a fuzzy supra β-closed set in X and $g^c \geq f^c\}$

$= \beta cl^*(f^c)$

2)

$\beta cl^*(f) = \wedge \{ g / g$ is a fuzzy supra β-closed set in X and $f \leq g\}$

$[\beta cl^*(f)]^c = 1 - \wedge \{ g / g$ is a fuzzy supra β-closed set in X and $f \leq g\}$

$= v \{g^c / g^c$ is a fuzzy supra β-open set in X and $f^c \geq g^c\}$

$= \beta int^*(f^c)$

Theorem: 3.9

For any fuzzy sets f and g in fuzzy supra topological space (X, δ^*)

1) $\beta int^*(f)\ v\ \beta int^*(g) = \beta int^*(f\ v\ g)$
2) $\beta cl^*(f) \wedge \beta cl^*(g) = \beta cl^*(f \wedge g)$

Proof:

1) Let $\beta int^*(f) = v \{h / h$ is a fuzzy supra β-open set in X and $h \leq f\}$

$\beta int^*(g) = v \{ k / k$ is a fuzzy supra β-open set in X and $k \leq g\}$

then the union of these sets

$\beta int^*(f)\ v\ \beta int^*(g) = v \{ h\ v\ k / h\ v\ k$ is a fuzzy supra β-open set in X and $h\ v\ k \leq f\ v\ g\}$

(if $j = h\ v\ k$)

$= v \{j / j$ is a fuzzy supra β-open set in X and $j \leq f\ v\ g\}$

$= \beta int^*(f\ v\ g)$

2) Let $\beta cl^*(f) = \wedge \{ h / h$ is a fuzzy supra β-closed set in X and $f \leq h\}$

$\beta cl^*(g) = \wedge \{ k / k$ is a fuzzy supra β-closed set in X and $g \leq k\}$

then the intersection of these sets

$\beta cl^*(f) \wedge \beta cl^*(g) = \wedge \{ h \wedge k / h \wedge k$ is a fuzzy supra β-closed set in X and $f \wedge g \leq h \wedge k \}$

(if $j = h \wedge k$)

$= \wedge \{j / j$ is a fuzzy supra β- closed set in X and $f \wedge g \leq j \}$

$= \beta cl^*(f \wedge g)$

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